# Heat Transfer Enhancement in Cool-thermal Discharge Systems from Ice Melting with Producing Chilled Air under Time-velocity Variations and External Recycles

Chii-Dong Ho*, Ho-Ming Yeh, Jr-Wei Tu and Ying-Sian Su

*Department of Chemical and Materials Engineering, Tamkang University*
*Tamsui, Taiwan 251, R.O.C.*

## Abstract

A mathematical model of cool-thermal discharge systems with external recycle by melting ice with producing chilled air to supply on-peak daytime air-conditioning needs was developed and studied analytically. The estimation of the outlet chilled air temperature and air mass velocity with convective transfer on free water surface under air time-velocity variations for specified inlet temperatures was illustrated. Theoretical results show that the recycle effect can effectively enhance the heat transfer efficiency compared with that in the device without recycle. Three numerical examples of ambient air temperatures varied with time in practical systems were simulated, and the result of air mass velocity with air traversed length as a parameter was also delineated.

***Key Words*:** Heat Transfer, Cool Thermal Discharge, Chilled Air, External Recycles

## 1. Introduction

The ability to help to match the electric utility supply and demand patterns in energy saving is a recyclic operation to make ice during nighttime peak, as referred to cool-thermal storage systems [1,2], and produce chilled air from melting ice during daytime peak to supply air-conditioning needs, as referred to cool-thermal discharge systems [3,4]. The high effective utilization at excess capacities of electric utility contributes to improved load management while the low cheaper rate offered by utility companies makes good economic sense during off-peak hours (typically 9 pm – 9 am) of ice making in cool-thermal charging systems.

The recycle effect is of importance in designing the heat and mass transfer due to the forced convection increment and widely used in absorption, fermentation, and polymerization, such as air-lift reactors [5] and draft-tube bubble columns [6], or loop reactors [7]. In conjunction with our previous works [8], this article proposes the recycle effect and time-variation of flowing air *in situ* contact in ice melts with specified outlet chilled air temperature in operating a cool-thermal discharge system. In this study, the theoretical formulation of the heat convection-conduction system in the presence of a moving liquid-solid interface [9–11] with convection transfer on the free water surface was developed using heat transfer and thermodynamic relations. The velocity of flowing air is controlled to produce the desired outlet chilled air temperature and the recycle-effect concept is introduced to enhance the heat transfer improvement for the practical demand in our daily life.

*Corresponding author. E-mail: cdho@mail.tku.edu.tw

## 2. Temperature Distribution in the Water Layer

An alternative arrangement with external recycle for producing chilled air with desired outlet temperature is to adjust the inlet air mass velocity, as shown in Figure 1. Before entering the open conduit, the fresh ambient air of mass velocity $G$ will premix the recycle mass velocity

$RG$ and outlet temperature $T_{f,o}$ regulated by using a conventional pump. Under this design condition, the ambient air with time-variation velocity was pumped through a parallel-plate channel of width $B$, length $L$, and height

## Nomenclature

| | | | |
|---|---|---|---|
| $a$ | defined by Eq. (6) | $P_0$ | power consumption of the device without recycle, *hp* |
| $B$ | width of conduit, m | Pr | Prandtl number ($\mu_f c_f / k_f$) |
| $b$ | defined by Eq. (6) | $Q_m$ | latent heat of ice melting, $kJ / kg$ |
| $c_f$ | specific heat of air, $J / kg \cdot K$ | $q$ | the heat transfer rate absorbed at the free water surface, *hp* |
| $c_w$ | specific heat of water, $J / kg \cdot K$ | $R$ | recycle ratio |
| $D_e$ | equivalent diameter of air channel ($2W$), $m$ | Re | Reynolds number ($D_e G/\mu_f$) |
| $f$ | friction factor | $T_{f,i}$ | inlet air temperature, $K$ |
| $G$ | air mass velocity, $kg / m^2 \cdot h$ | $T^*_{f,i}$ | mixed air temperature at inlet, $K$ |
| $h_m$ | average convective heat transfer coefficient, $W / m^2 \cdot K$ | $T_{f,m}$ | arithmetic-mean temperature of air, $K$ |
| $I_h$ | heat transfer improvement, defined by Eq. (19) | $T_{f,o}$ | outlet air temperature, $K$ |
| $I_p$ | power consumption increment, defined by Eq. (21) | $T_p$ | melting point of ice, $K$ |
| $k_f$ | thermal conductivity of air, $W / m \cdot K$ | $T_s$ | temperature at water free surface, $K$ |
| $k_w$ | thermal conductivity of water, $W / m \cdot K$ | $T$ | time, $h$ |
| $L$ | traversed length of air, $m$ | $V$ | flowing air velocity, $m / s$ |
| $\ell w_f$ | friction loss, $kJ / kg$ | $V_0$ | flowing air velocity of the device without recycle, $m / s$ |
| $\ell w_{f,0}$ | friction loss of the device without recycle, $kJ / kg$ | $W$ | distance between water surface and the insulated plate, $m$ |
| $Nu_m$ | average Nusselt number ($h_m D_e/k_f$) | $X$ | thickness of melting ice, $m$ |
| $p$ | atmosphere pressure, atm | $x$ | $x$-axis, $m$ |
| $P$ | power consumption, *hp* | | |

## Greek letters

| | | | |
|---|---|---|---|
| $\alpha_w$ | thermal diffusivity of water, $m^2 / h$ | $\rho_w$ | density of water, $kg / m^3$ |
| $\mu_f$ | air viscosity, $kg / m \cdot s$ | $\psi$ | excess temperature, $T$-$T_p$, $K$ |
| $\rho_f$ | density of air, $kg / m^3$ | $\psi_s$ | excess temperature at water free surface, $K$ |

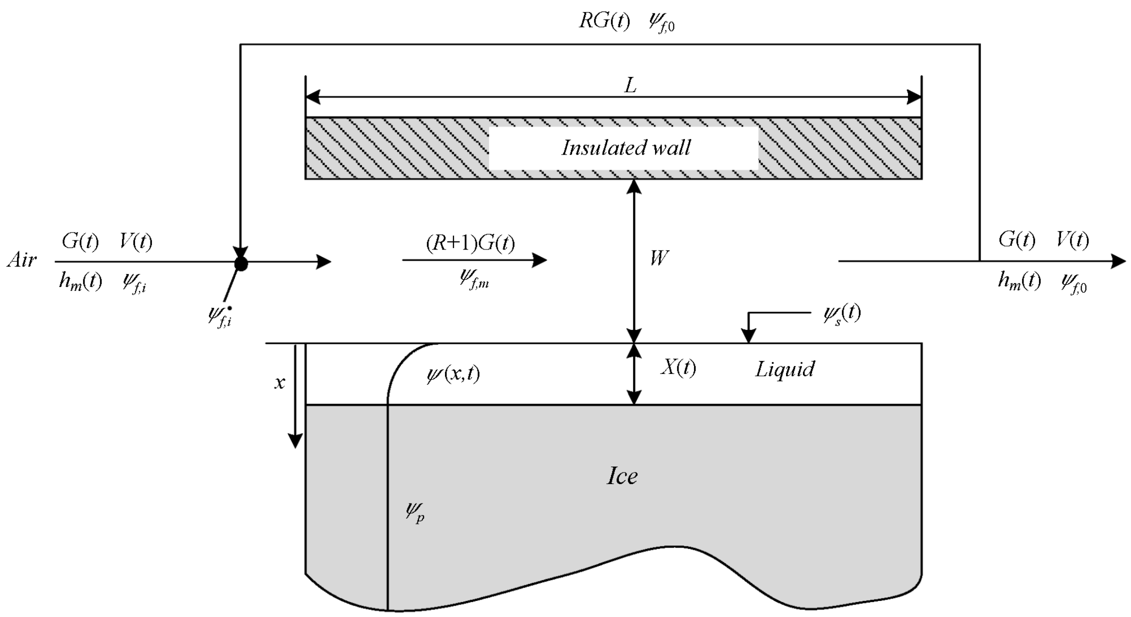


**Figure 1.** Schematic diagram of a cool-thermal discharge system with external recycle.

$W(W/B << 1)$ and chilled by a semi-infinite ice layer with uniform temperature initially.

With the assumption of neglecting the volume change during ice melting process, the heat transfer equation in terms of $\psi(x,t)$ was obtained with introducing an excess temperature $\psi = T - T_p$ within the water region

$$\partial\psi / \partial t = \alpha_w (\partial^2\psi / \partial x^2), \quad 0 \le x \le X(t) \tag{1}$$

The initial and boundary conditions are

$$X(t) = 0, \text{ at } t = 0; \qquad \text{for IC} \tag{2}$$

$$-k_w \frac{\partial\psi}{\partial x} = h_m(\psi_{f,m} - \psi_s) \text{ at } x = 0; \qquad \text{for BC1} \tag{3}$$

$$\psi(x,t) = 0, \text{ at } x = X(t); \qquad \text{for BC2} \tag{4}$$

$$-k_w\left(\frac{\partial\psi}{\partial x}\right) = \rho_w Q_m\left(\frac{dX}{dt}\right), \text{ at } x = X(t); \text{ for BC3} \tag{5}$$

where $\psi_{f,m} = T_{f,m} - T_p$ and $X(t)$ is the melted ice thickness. $T_{f,m}$ infers the arithmetic-mean air temperature at the free water surface, and $h_m(\psi_{f,m} - \psi_s)$ denotes the flux of energy absorption or the cool-thermal discharge flux at the free water surface and varies hourly with air time-variations velocity during the discharging period.

By following the same mathematical treatment performed in the previous work [4], the temperature distribution in water layer and thickness of melted ice were obtained by solving Eq. (1) analytically with the use of Eqs. (2)–(5). The temperature distribution in the water layer was obtained as follows:

$$\frac{\psi(x,t)}{Q_m/c_w} = a\left(\frac{x}{X} - 1\right) + b\left(\frac{x}{X} - 1\right)^2 \tag{6}$$

where

$$a = -\sqrt{2b} = \frac{k_w + h_m X}{2(k_w + h_m X/2)}\left[1 - (1 + 4\mu)^{1/2}\right] \tag{7}$$

$$\mu = c_w X h_m \psi_{f,m}(k_w + h_m X/2) \big/ Q_m(k_w + h_m X)^2 \tag{8}$$

and

$$\left(-\frac{aQ_m}{2c_w} + \frac{a^2 Q_m}{6c_w} + \frac{Q_m}{c_w}\right)\frac{dX}{dt} = -\frac{\alpha_w h_m}{k_w}\frac{a^2 Q_m}{2c_w} + \frac{\alpha_w h_m}{k_w}\frac{aQ_m}{c_w} + \frac{\alpha_w h_m}{k_w}\psi_{f,m} \tag{9}$$

The temperature-time history at the free surface $x = 0$ was obtained in the form

$$\psi_s/(Q_m/c_w) = (T_s - T_p)/(Q_m/c_w) = -a + b \tag{10}$$

## 3. The Average Convective Heat-transfer Coefficient and Transfer Efficiency Improvement

Let $T_{f,i}$, $T_{f,o}$ and $T_{f,m}$ respectively denote the inlet, outlet and arithmetic-mean temperatures of the air of a cool-thermal discharge system. The average convective heat-transfer rate between flowing air and the water surface was estimated according to the air arithmetic-mean temperature. The heat transfer rate absorbed at the free water surface may be expressed as

$$q(t) = BLh_m[T_{f,m} - T_s] \tag{11}$$

$$= G(t)c_f(BW)(T_{f,i} - T_{f,o}), \tag{12}$$

in which

$$T_{f,m} = (T^*_{f,i} + T_{f,o})/2 \tag{13}$$

and

$$T^*_{f,i} = (T_{f,i} + RT_{f,o})/(R+1) \tag{14}$$

where $h_m$ and $c_f$ denote the average convective heat-transfer coefficient and specific heat of air, respectively, while $G$ is the air mass velocity. The average convective heat-transfer coefficient of the flowing air can be obtained from Eqs. (11) and (12) as:

$$h_m = \frac{2W(R+1)G(t)c_f(T_{f,i} - T_{f,o})}{L[T_{f,i} + (2R+1)(T_{f,o}) - 2(R+1)T_s)]} \tag{15}$$

The empirical expression for the average convective heat transfer coefficient, $h_m$, was given by Mercer *et al.* [12] and Kays and Crawford [13] for laminar and turbulent flow as shown Eqs. (16) and (17), respectively, while the forced-convection flow between two parallel plates with insulation on one side. The average Nusselt number of fully developed laminar flow with $(\mathrm{RePr}D_e/L) < 1{,}000$ [12] is:

$$Nu_m = \frac{h_m D_e}{k_f} = 4.9 + \frac{0.0606(\mathrm{Re}\,\mathrm{Pr}\,D_e / L)^{1.2}}{1 + 0.0909(\mathrm{Re}\,\mathrm{Pr}\,D_e / L)^{0.7}\,\mathrm{Pr}^{0.17}},$$
$$\mathrm{Re} < 2100, \quad 0.1 < \mathrm{Pr} < 10 \tag{16}$$

where $D_e$ denotes the equivalent diameter of the air passage and is equal to four times the cross-sectional area of the duct divided by the wetted perimeter, $D_e = 2W$. Re and Pr are the Reynolds and Prandtl numbers, respectively. The correlation average Nusselt number for fully developed turbulent flow is as follows [13]:

$$Nu_m = \frac{h_m D_e}{k_f} = 0.0185\,\mathrm{Re}^{0.8}, \quad \mathrm{Re} > 2100. \tag{17}$$

where $\mathrm{Re} = D_e V \rho_f / \mu_f = D_e G / \mu_f$.. One may evaluate the average convection heat transfer coefficient by Eqs. (16) and (17) for laminar and turbulent flow, respectively, or by Eq. (15) directly. Then the average Nusselt number can be expressed as follows:

$$Nu_m = \frac{h_m D_e}{k_f} = \frac{2W(R+1)G(t)c_f(T_{f,i} - T_{f,o})D_e}{L[T_{f,i} + (2R+1)T_{f,o} - (2R+1)T_s]k_f} \tag{18}$$

The heat-transfer efficiency improvement of cool-thermal discharge devices with external recycle is best illustrated by calculating the percentage increase in heat transfer rate, based on the device without recycle, as

$$I_h = \frac{Nu_m - Nu_{m,0}}{Nu_{m,0}} = \frac{(R+1)[T_{f,i} + T_{f,o} - 2T_s]}{[T_{f,i} + (2R+1)T_{f,o} - 2(R+1)T_s]} - 1 \tag{19}$$

## 4. Power Consumption Increment

The friction loss in conduits may be estimated by

$$\ell w_f = \frac{2 f V^2 L}{g_c D_e} \tag{20}$$

where $V$ and $D_e$ denote the average velocities of air in the conduits and the equivalent diameters of the conduits, respectively, while the friction factor $f = 0.079\,\mathrm{Re}^{-0.25}$ for the special case of a hydraulically smooth surface with $2{,}100 < \mathrm{Re} < 100{,}000$, known as the Blasius equation, and $f = 24/\mathrm{Re}$ for the laminar flow in the parallel-plate conduit, in which $\mathrm{Re} = D_e V \rho_f / \mu_f = D_e G / \mu_f$.

The friction losses in the conduit of the device without recycle were calculated by $P_0 = 48 V_0^2 \mu_f WBL / g_c D_e^2$ and $P_0 = 0.1582 V_0^{2.75} \mu_f^{0.25} \rho_f^{0.75} WBL / g_c D_e^{1.25}$ for laminar and turbulent flows, respectively and those of the device with recycle are $P = 48[(R+1)V]^2 \mu_f WBL / g_c D_e^2$ and $P = 0.1582[(R+1)V]^{2.75} \mu_f^{0.25} \rho_f^{0.75} WBL / g_c D_e^{1.25}$. As an illustration, the power consumption of the device without recycle was estimated by the working dimensions as follows: $B = 4$ m, $W = 0.1$ m, $L = 10$ m, 20 m and 30 m, $\mu_f = 1.82 \times 10^{-5}$ kg/m·s, and $\rho_f = 1.207$ kg/m$^3$. Under these working dimensions, the theoretical predictions show that the flow is turbulent. Accordingly, the power consumption increment $I_p$ due to the friction losses ($\ell w_f$ for the device with recycle while $\ell w_{f,0}$ for the device without recycle) can be readily derived as

$$I_p = \frac{P - P_0}{P_0} = \frac{[(R+1)V]\ell w_f - V_0 \ell w_{f,0}}{V_0 \ell w_{f,0}}$$
$$= \frac{\{0.1582[(R+1)V]^{2.75} \mu_f^{0.25} \rho_f^{0.75} WBL / g_c D_e^{1.25}\} - \{0.1582 V_0^{2.75} \mu_f^{0.25} \rho_f^{0.75} WBL / g_c D_e^{1.25}\}}{0.1582 V_0^{2.75} \mu_f^{0.25} \rho_f^{0.75} WBL / g_c D_e^{1.25}}$$
$$= \frac{[(R+1)V]^{2.75} - [V_0]^{2.75}}{[V_0]^{2.75}} \tag{21}$$

## 5. Numerical Examples

Let $t_0$ be the total discharge time in one-day operations. Three cases represent the practical simulation of the ambient temperature distribution for one day. The Case 1 shows the constant ambient temperature case and Case 2 and Case 3 are the cases of ambient temperature varied with time from sunrise to sunset.

**Case 1.** $T_{f,i}^1 = 305$ (constant), (22)

**Case 2.** $T_{f,i}^2 = 304 + 2\sin(\pi t / t_0)$, (23)

**Case 3.** $T_{f,i}^3 = 303 + 5\sin(\pi t / t_0)$. (24)

As an illustration, we assign the following numerical values: $T_p = 273$ K, $Q_m = 334$ kJ/kg and $t_0 = 10$ h. The physical properties of air at 1 atm and 20 °C are [12]: $\rho_f = 1.207$ kg/m$^3$, $c_f = 1004.8$ J/kgK, $k_f = 2.57 \times 10^{-2}$ W/mK, $\mu_f = 1.82 \times 10^{-5}$ kg/m·s, and Pr = 0.71. The Reynolds numbers were checked using Eq. (18) with the traversed length of air and the air mass velocity.

## 6. Results and Discussion

The temperature at the free water surface, $T_s$, was obtained by Eq. (10) and the results of Case 1 were shown in Figures 2 and 3 with the recycle ratio and specified outlet chilled air temperature as parameters, respectively. The temperature at the free water surface increases with the recycle ratio and traversed length of air, as shown in Figure 2, and increases with increasing the specified outlet chilled air temperature as indicated in Figure 3.

The time-variation of air mass velocity during operating period is presented in Figure 4 with the traversed length of air and recycle ratio as parameters. The air mass velocity increases with increasing the traversed length of air and recycle ratio but decreases as the discharging process proceeds for Case 1. Figure 5 shows that the air velocity varies with operating time to meet the outlet chilled air temperature for Case 1, say $T_{f,o}$ = 27 °C, $T_{f,o}$ = 26 °C and $T_{f,o}$ = 25 °C, respectively, while Figure 6 presents the air velocity for three cases with recycle ratio as a parameter. The required air velocity increases with increasing the specified outlet chilled air temperature, as observed from Figure 5. The air velocity increases with recycle ratio for all of the three cases, it is seen from Figure 6 that the air

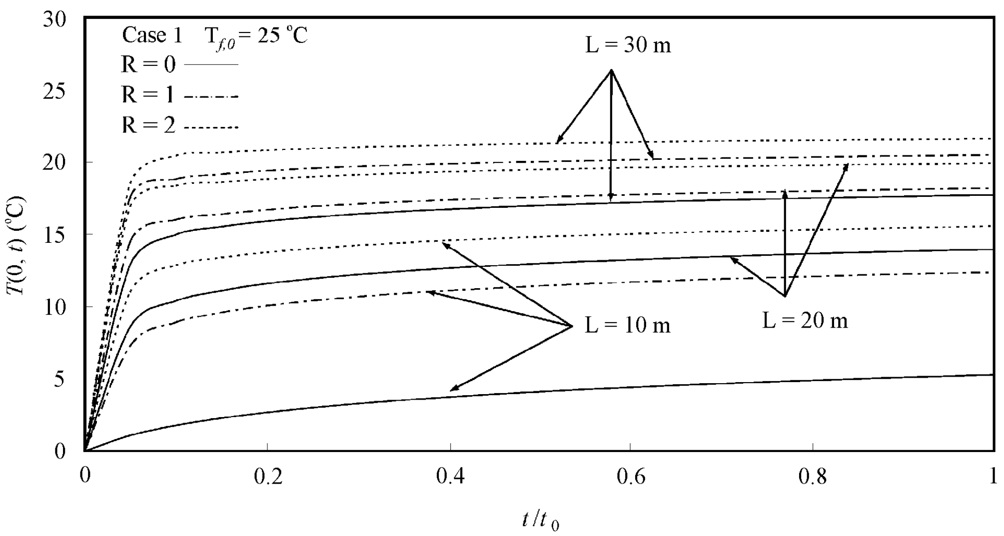


**Figure 2.** Temperature-time history at the free surface with the recycle ratio as a parameter for Case 1.

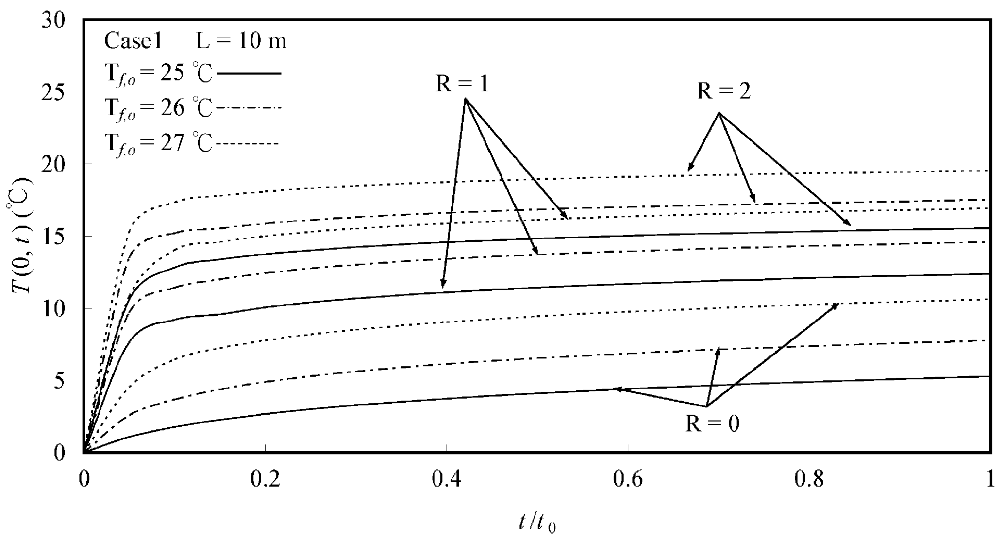


**Figure 3.** Temperature-time history at the free surface with the outlet temperature as a parameter for Case 1.

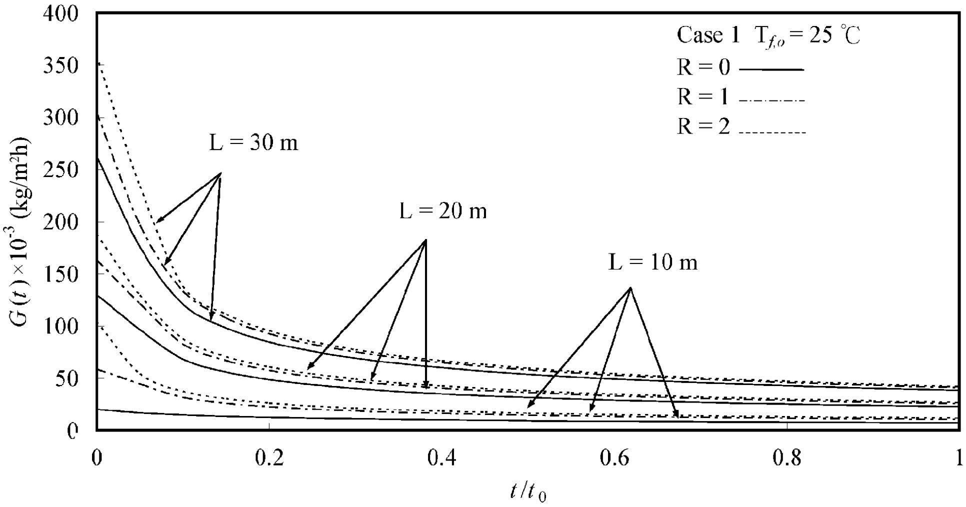


**Figure 4.** The air mass velocity with the traversed length of air as a parameter for Case 1.

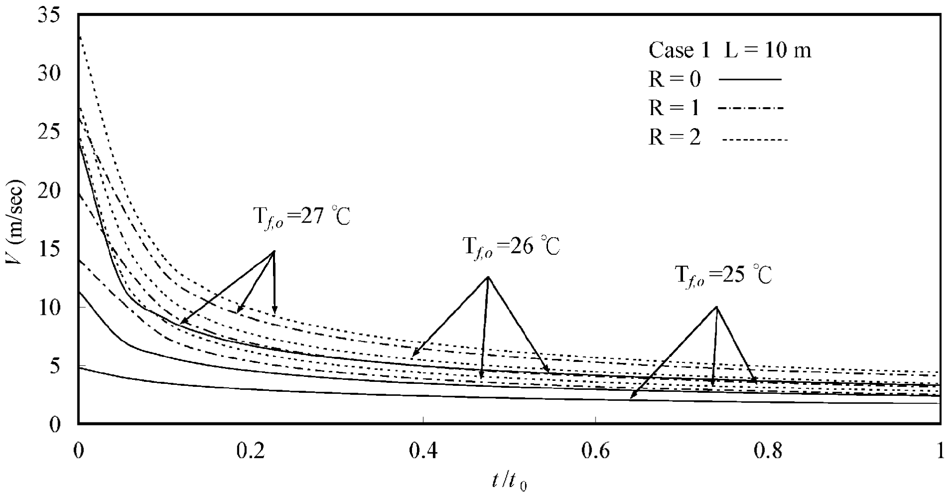


**Figure 5.** The time-variation velocity of air with the outlet temperature as a parameter for Case 1.

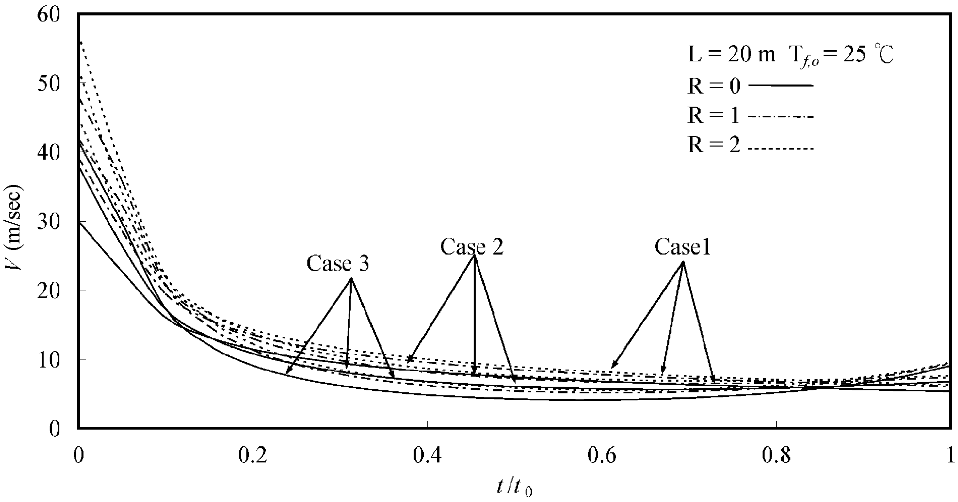


**Figure 6.** The time-variation velocity of air for three cases.

velocity decreases with the operating time for Case 1, however, there exists minimum air velocity at $t/t_0 = 0.7$ and $t/t_0 = 0.6$ for Cases 2 and 3, respectively. The heat transfer rate absorbed at the free water surface was calculated by Eq. (11) or Eq. (12) and the results were shown in Figure 7 with recycle ratio as a parameter. The heat transfer rate increases with the recycle ratio and traversed length of air as observed from Figure 7. The heat-transfer efficiency improvement of the cool-thermal discharge devices with external recycle was estimated by Eq. (19) based on that of the device without recycle. The heat-transfer efficiency improvement was shown in Figure 8 with the outlet temperature as a parameter for Case 1. The values of $I_h$ increase with the outlet chilled air temperature, especially for large recycle ratio. The minimum air velocity in this study is 1.74 m/s for Case 1, L = 10 m, R = 0 and $T_{f,o}$ = 25 °C as shown in Figure 5, and the corresponding Reynolds number with the working dimensions is Re = 23,079, which is in the turbulent region. Therefore, the power consumption increment due to the external-recycle operation was determined by the Eq. (21). Table 1 shows that the power consumption increment increases with the recycle ratio and decreases with the traversed length of air and operating time.

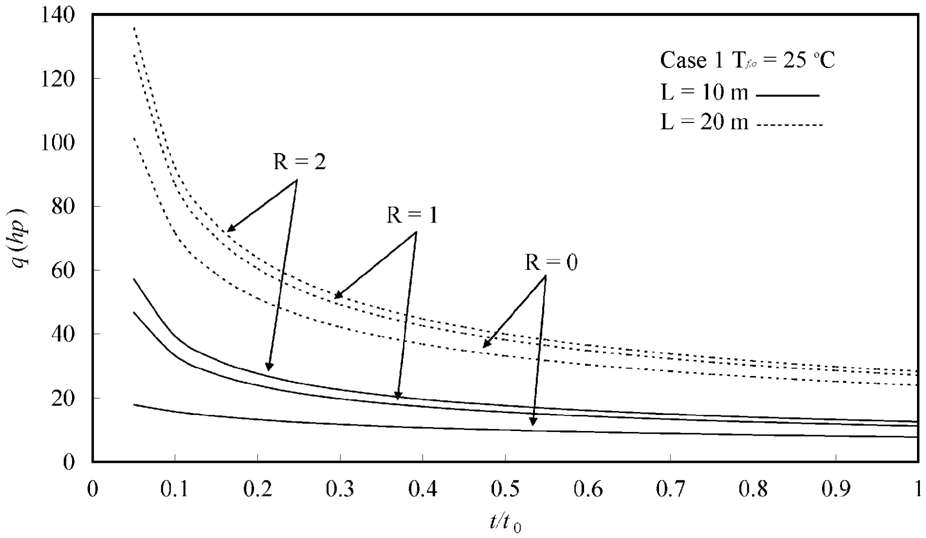


**Figure 7.** The values of $q$ with the specified outlet chilled air temperature as a parameter for Case 1.

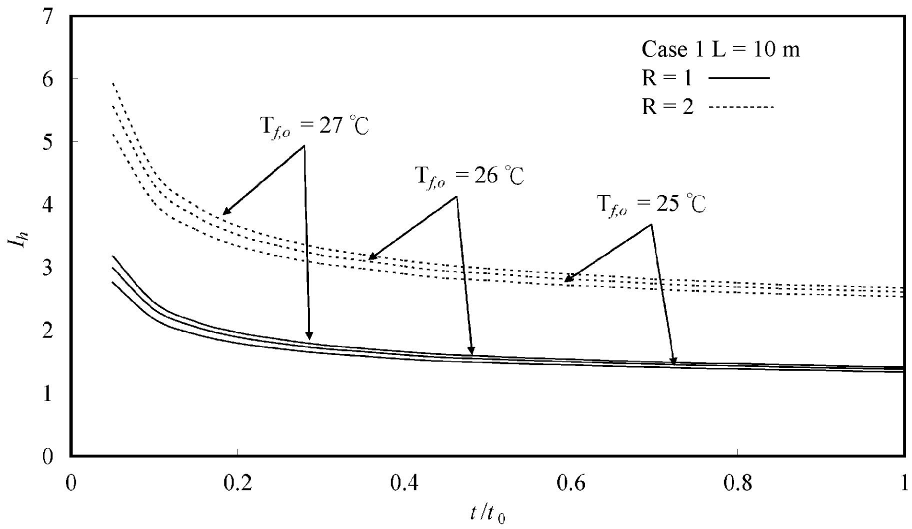


**Figure 8.** The heat-transfer efficiency improvement $I_h$ vs. $t/t_0$ for Case 1with the outlet chilled air temperature as a parameter.

**Table 1.** The power consumption increment of Case 1 with recycle ratio as a parameter

| $I_p$ | L = 10 | | L = 20 | |
|---|---|---|---|---|
| $t$ | R = 1 | R = 2 | R = 1 | R = 2 |
| 0 | 127.04 | 1885.10 | 13.04 | 61.57 |
| 0.2 | 33.26 | 154.33 | 9.64 | 36.71 |
| 0.4 | 23.85 | 106.29 | 9.06 | 34.13 |
| 0.6 | 20.71 | 90.17 | 8.77 | 32.96 |
| 0.8 | 18.91 | 81.14 | 8.55 | 31.94 |
| 1 | 17.63 | 75.65 | 8.42 | 31.39 |

## 7. Conclusions

The theoretical study of cool-thermal discharge systems with adjustable air mass velocity from ice melting to control the outlet chilled air temperature was developed with the energy balance analysis within the air flowing channel. The required air velocity to meet the specified outlet chilled air temperature with the design parameter (traversed length) and operating parameters (outlet chilled air temperature and recycle ratio) for three inlet ambient temperatures was calculated. The average convection heat-transfer coefficient and air mass velocity were estimated from Eqs. (15) and (18), respectively, while the temperature at the free water surface was evaluated from Eq. (10). The influence of recycle effect on the device performance was discussed. Considerable heat-transfer efficiency improvement is obtained by employing the new device with external recycle instead of using the device without external recycle.

Conclusively, according to both Figures 7 and 8, the heat transfer rate can be significantly improved by the external recycle operation especially for high outlet chilled air temperature and large traversed length of air. One may follow the mathematical formulations performed in the present paper to the conduction-convection transfer problems associated with a moving liquid-solid interface and external recycle.

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